The Andrew and Erna Viterbi Faculty of
ELECTRICAL & COMPUTER ENGINEERING

SIPL
Signal and Image Processing Lab

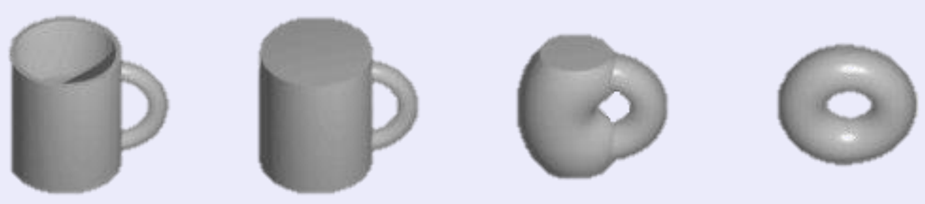
TECHNION
Israel Institute of Technology

Topology of Signals

Saar Ron and Tomer Fireaizen, Supervised by Prof. Omer Bobrowski

Introduction

- Topology deals with the properties of a geometric object, that are preserved under continuous deformations



- Topological Signal Processing is a new and exciting field in the Signal Processing world
- It is based on converting a signal to a point cloud and measuring its topology
- This work shows a novel solution to a real-world problem using topological tools

Goals

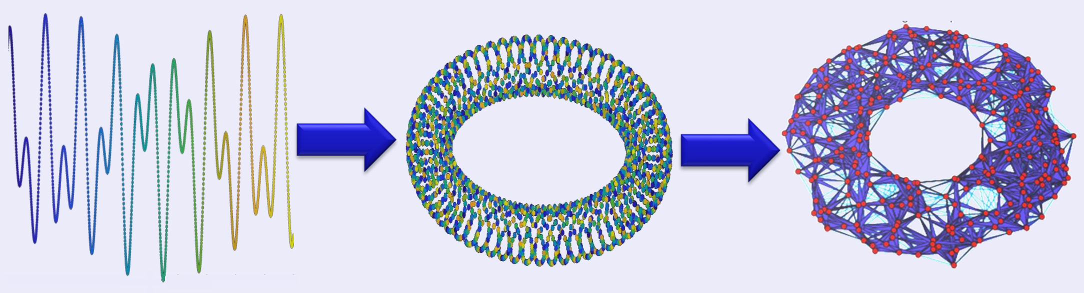
- Learn the field of Topological Signal Analysis
- Improve the current methods of topological features extraction
- Apply to a real-world problem

Challenges

- Limited number of previous works, Mostly theoretical
- Hard and Complicated Math
- Low SNR conditions

Signal to Topological Object

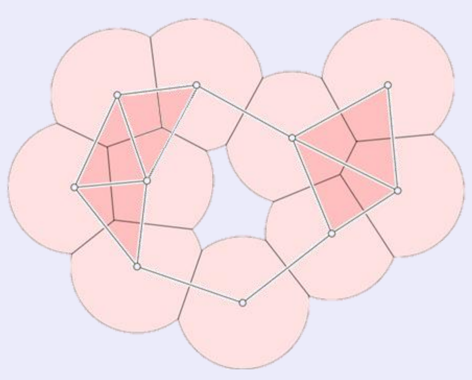
- Signal ➡ Point Cloud



- Using Sliding Window transformation:

$$\mathcal{T}_{\tau,w}(f[n]) = [f[n], f[n+\tau], \dots, f[n+w\tau]] \in \mathbb{R}^{w+1}$$

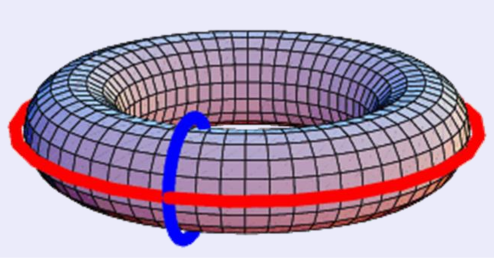
- Point Clouds converted to Simplicial Complexes using the Alpha Complex method



Topological Properties

- Zero order cycle – connected component
- First order cycle – hole (unremovable loop)
- Second order cycle – air pocket
- Higher degrees exist

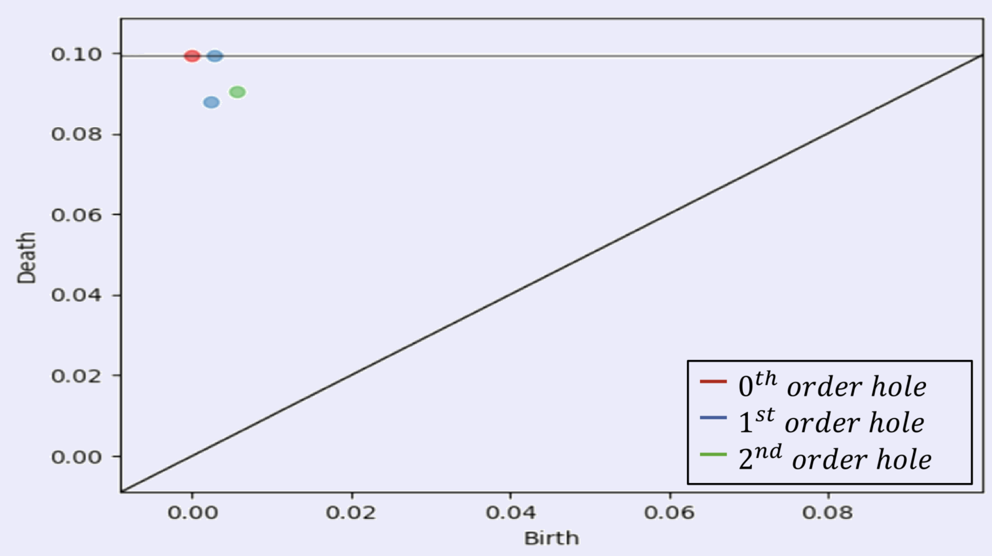
Torus



#Connected Components = 1
#Holes = 2
#Air Pockets = 1

Cycle Diagram

- Alpha complex process defines birth and death times of cycles
- These times can be represented in a diagram:



Feature Extraction

- Features can be extracted from the diagram:
- $\mathcal{H}_d$ - The set of all holes of dimension d:

$$\mathcal{H}_d = \{\forall h | h \text{ is in dimension } d\}$$

- $\ell(h)$ - The life span of h: $death(h) - birth(h)$
- $\mathcal{L}_d^i$ - The hole from dimension d with the i^{th} longest life span

$$\mathcal{L}_d^i = i^{th} \max_{h \in \mathcal{H}_d} (\ell(h))$$

- $|\mathcal{H}_d|$ - Cardinality of $\mathcal{H}_d$
- Examples of statistical features:

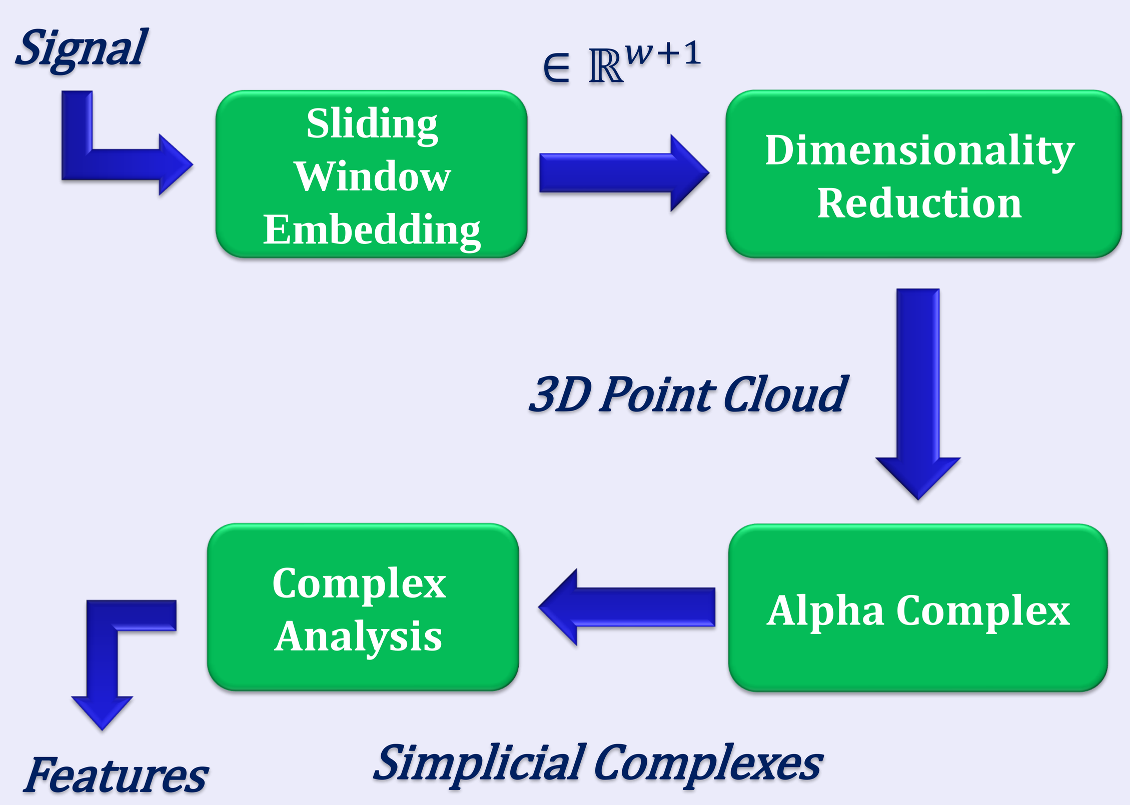
$$\frac{\mathcal{L}_1^1}{|\mathcal{H}_1|}, std_i(\mathcal{L}_d^i), |\mathcal{H}_1| \text{ when } \ell_1 > T, \dots$$

- Examples of shape-related features:

Periodic Score: $PS = \mathcal{L}_1^1 - \mathcal{L}_1^2$

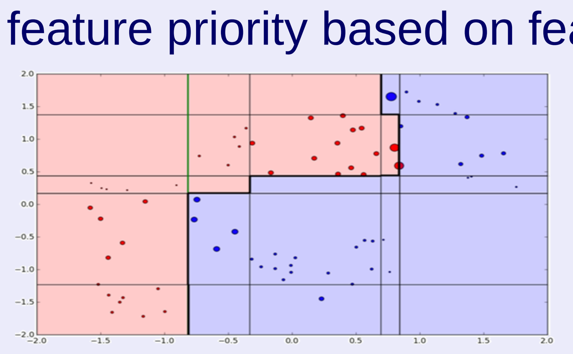
Quasi-Periodic Score: $QPS = \mathcal{L}_1^1 \times \mathcal{L}_1^2 \times \mathcal{L}_2^1$

Analysis Scheme

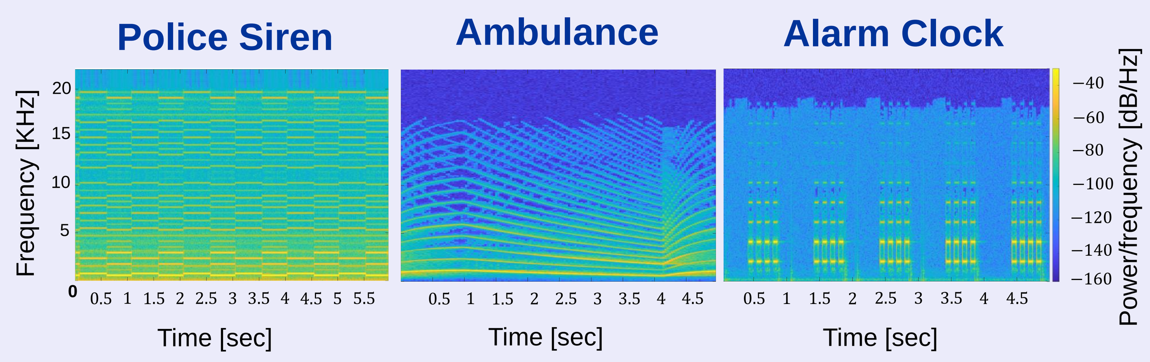


AdaBoost Classifier

- This classifier can show the strength of topological features
- Modified feature priority based on feature ranking



Alarm Detection

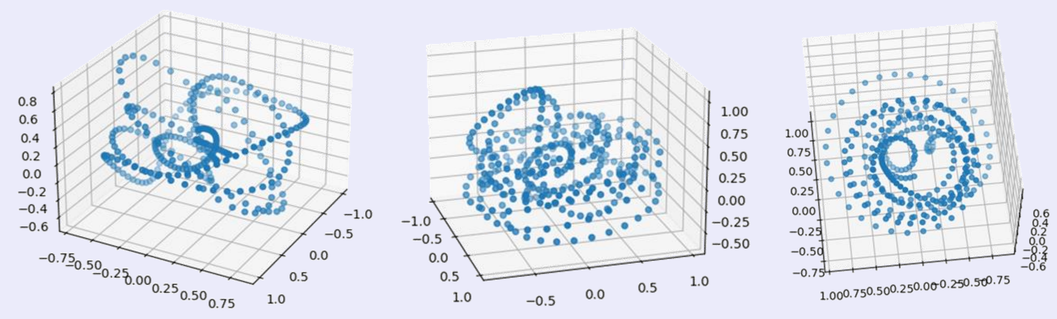


- Alarms are mathematically ill-defined
- However, alarms tend to be annoying since they are composed of quasi-periodic signals
- Topological Signal Processing is excelling in the detection of quasiperiodicity

Dataset – UrbanSound8K

- State-of-the-art dataset of urban sounds
- Contains ~8,700 excerpts from 10 classes:
  - ~900 alarms
  - ~7,800 other sounds like:

    Car engine, Dog barks, Gun shots, ...
- Incremental learning on balanced batches



Dog Barks Traffic Alarm

Results

- Previous works used deep learning networks
- Spectrograms were used as inputs
- Our work uses a classic classifier

Name	Year	Accuracy	False Negative	False Positive
Zhang et al.	2018	96.4%	17.33%	1.96%
Garg et al.	2020	96.7%	15.76%	1.96%
Li et al.	2021	98.7%	5%	0.95%
Ours	2021	**98.8%**	7.14%	0.39%

Run Time Analysis

- ~1 [sec] to classify four windows of 4 [sec]
- Implemented in Python and running on CPU

Conclusion

- A State-of-the-art AdaBoost classifier based on topological features for alarm detection
- Introducing many contributing topological features for signal analysis
- Feasible to run in real-time
- A proof of concept for a real-world application of Topological Signal Processing

June 2021